

Report on research activity while holding the ISRP award

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1 Introduction

This report summarizes my research in the period when I was a recipient of an ISRP award, namely from 08/01/01 till 08/31/02. The focus of this research was the study of computer networks. The work was carried out in collaboration with Ernie Barany [1], [2]. The article [1] is attached to this report (work on [2] is still in progress). We have developed a general approach to modeling dynamics of computer networks, which we used to analyze local area networks (LAN's) functioning according to the CSMA/CD protocol. As we indicate in the report our approach is applicable in a much wider context. We take advantage of some structures and properties (some of which are present only in an approximate sense) which were not exploited previously. These properties lead to a significant simplification in the analysis. In the course of this report we will explain our approach to modeling and make a comparison with the classical approach.

In our work we have addressed the question of congestion of a network, which, roughly speaking, occurs if the number of information packets produced exceeds the throughput of the network. We have been able to compute stability coefficients corresponding to maximal rates of packet generation for which the network does not become congested. These computations are based on the modeling simplifications mentioned above and are partially theoretical and partially numerical.

2 The model of a computer network

We have developed our approach using a model of a local area network (LAN) as an example. The model we consider consists of n nodes, each having a buffer of infinite length. The infinite buffer assumption is one of our important idealizations which allows us to reduce the dynamics of one node of the network to a one dimensional random walk (this point will be explained in more detail in the sequel). We assume that the nodes generate information packets with fixed rate and attempt to send a packet in every time slot as long as they are “allowed” to do so. This assumption of a busy network is our second important simplification. The transmission of information out of the LAN is governed by the so called CSMA/CD protocol which works in the following way. In the beginning of every time slot each node has a waiting time, which is a non-negative integer. If the waiting time is 0 then the corresponding node is allowed to send a packet and does so, according to our “busy network” assumption. If two nodes attempt a transmission simultaneously then a collision occurs and both nodes are assigned new waiting times according to some formula.

3 Analysis of the model

We use Markov methodology to analyze the model. Our basic random variables are the queue lengths at each node. In other words we consider a vector random variable whose components are queue lengths at each node. The “busy network” assumption makes it possible to represent queue lengths as functions of the waiting times. This is clear, since a packet is sent if and only if the waiting time of a node is 0. In other words, the number of packets transmitted by a node in a time interval is completely determined by the histogram of this node’s waiting time, while packet generation is independent of other dynamics. It is therefore possible to analyze packet transmission separately from packet generation, which is an important decoupling, allowing for a significant simplification of the analysis. Using this approach it is very profitable to use the vector of waiting times as the Markov variable. It is easy to check that this Markov chain is ergodic and therefore there is a unique stationary distribution and many useful properties hold, in particular the ergodic theorem. Using the ergodic theorem we can compute the asymptotic distribution of the two lowest waiting times and then use this

distribution to compute the *throughput*, i.e. the limiting value of the average number of packets sent out by a node. If the rate of packet generation is higher than throughput the node will become congested, i.e. the queue will grow unboundedly and otherwise its queue will stay bounded.

The way we obtain the above described stability result is by observing that, asymptotically (that if the stationary distribution of the two lowest waiting times is used to describe packet transmission), the evolution of the queue is a one dimensional random walk. If throughput is larger than the generation rate than the drift of the walk is to the right, i.e. the queues increase unboundedly. If throughput is less than the generation rate then the drift is to the left and the queues decrease. Clearly, near 0 this model is inaccurate, since the queues cannot be negative. In other words, the busy network assumption only makes sense if there are sufficiently many packets. Hence negative drift only means that the queues remain bounded.

It is worth pointing out that our analysis is easily adaptable to different algorithms for choosing waiting times. In [1] we used both the random draw from fixed interval and exponential back-off algorithms, but our approach is highly modular, so a study involving another algorithm could easily be carried out.

4 Traditional approach to modeling and analysis of CSMA/CD

We have found some articles where Markov modeling was used to model CSMA/CD type of networks [7], [6], [4], [5]. The modeling assumes that each node has a finite buffer and the queue length is the basic state variable for the modeling. Another random variable considered is the number of users with a given queue length (this is of course a vector variable). Stationary distributions are found when possible, otherwise equilibrium point analysis or numerical approaches are used. We have not seen the use of the ergodic theorem as basis for a numerical method. This is the approach that worked very well for us to derive the distribution of the two lowest waiting times as described in the previous section. More importantly, we have not seen the use of the busy network assumption. In our opinion an important point in the modeling should be to model packet generation and packet transmission and see how they are coupled. The case we have studied is when they are

not coupled at all, but most likely this approach is useful even if there is coupling. The reduction to a one dimensional random walk also seems new.

5 Applicability of our modeling in other settings

As pointed out by prof. van As from the Institute for Communication Networks, Vienna University for Technology, our modeling is applicable to a variety of networks in which data is queued according to some protocol. Since, as mentioned above, our approach is highly modular, it is a fairly simple exercise to adapt the model to a different queuing protocol. As an example we give a network studied by Benji and van As [3]. They consider a circular network consisting of nodes connected by a WDM optical cable. The cable is able to transmit information packets at many different frequencies. In this type of networks the access protocols do not involve packet collisions, yet modeling of traffic is similar to the classical CSMA/CD case. In networks of this type packets with different QoS (quality of service) specifications may be transmitted. The authors point out that these different types of data may require different access strategies. They also give detailed qualitative descriptions of the possible access protocols involving data types with different QoS classes. Yet their quantitative modeling involving Markov processes is restricted to a relatively simple situation, namely one data type, mainly due to the complexity of the problem. We are confident that using our modeling approach, using the busy network assumption, and the resulting decoupling of the dynamics we can analyze models involving transmission of different data types and complicated access protocols.

References

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