A Reduced Generator Model with Excitation Limits

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ABSTRACT
This paper presents a simplified generator model for use in dynamic studies of electric power systems. The proposed model captures important characteristics of generators that include voltage and frequency variation as well as excitation limits. The behavior of the model is compared to that of a well-accepted higher order model through application to an example two-bus power system. Examination of equilibria with or without excitation limits, responses to load variations, and responses to step inputs indicate that the proposed model captures key behaviors, and may often be sufficient for analysis of power system dynamics.

KEY WORDS
Model reduction, simulation, electric power, generator models

1. Introduction
The consequences of electric power outages dictate the importance of conducting power system studies. These studies will most likely be based upon models that combine representations of the various components (e.g., generators, loads, and network) in the power system. It is not possible to develop models that describe all phenomena at all time scales, especially in the context of feasible simulation of realistic power systems. All-encompassing models would require a large amount of parameter data, lack tractability for a thorough interpretation of results, and necessitate extensive computational resources. Thus, to perform computer simulation and analysis of power system responses simple models should be utilized that capture the attributes of interest.

Generator models of varying complexity are used in electric power system representations depending on the analysis objectives. In voltage stability studies where exciter limit events are of interest a generator model with excitation dynamics of varying sophistication is commonly used [1-4]. Simpler dynamic generator models with reactive power limits have been proposed by Dobson and Lu [5] and the IEEE/PES Power System Stability Subcommittee [6]. These models are similar in that they assume a fixed generator voltage until exciter limits are reached after which the limits are enforced while generator voltage is allowed to vary indicating a loss of voltage control. Thus far these models have been utilized for analytical and simulation studies of small systems with little consideration given towards how their characteristics compare to more common models such as those presented in [1, 2, 4].

In this paper it is shown that a new simplified generator model, similar to those presented in [5, 6] can be used to obtain similar dynamic responses and perform much of the same stability and control analyses as the higher order models used in the literature. Phenomenon such as voltage excitation and associated limits which directly influence power system stability [4-8] are included in this simplified model.

2. Test System and Model Formulation
A two-bus power system shown in figure 1 is chosen for the comparisons presented in this paper. The two bus example consists of one generator connected to a fixed load through a transmission line. Constant complex load power consumed at bus 2 is denoted by \( P_D^c + jQ_D^c \), power injected by the generator into bus 1 is \( P_G + jQ_G \), and \( V_1^{\angle \theta_1}, V_2^{\angle \theta_2} \) are the phasor representations of the bus voltages.

![Figure 1. Two-bus test system](image)

The algebraic power balance constraints for the system are given by:

\[
0 = P_G - V_I^2 G + V_I V_d (G \cos \theta - B \sin \theta) \tag{1a}
\]

\[
0 = Q_G - V_I^2 B + V_I V_d (G \sin \theta + B \cos \theta) \tag{1b}
\]

\[
0 = -V_I^2 G + V_I V_d (G \cos \theta + B \sin \theta) - P_D \tag{1c}
\]
\[ 0 = - V_j B - V_i V_j (G \sin \theta - B \cos \theta) - Q_d \quad (1d) \]

where (1a), (1b) represent real power and reactive power balance equations, respectively, at bus 1, (1c), (1d) represent real power and reactive power balance equations, respectively, at bus 2, \( \theta = \theta_1 - \theta_2 \), and the line admittance \( G + jB = I/(R + jX) \).

This paper considers a classical flux-decay model [1, 2] to which the proposed reduced model will be compared. Both models incorporate the swing equation and differ primarily in the voltage control and excitation dynamics. The following common representation of the swing equation captures the mechanical dynamics of the generator through rotor angle \( \delta \) and generator speed \( \omega \). Full details of the model can be found in [1-4].

\[ \dot{\delta} = \omega - \omega_{ref} \quad (2a) \]

\[ \dot{\omega} = \frac{1}{M} (P_n - P - D(\omega - \omega_{ref})) \quad (2b) \]

The flux-decay model adds the following voltage control and excitation dynamics

\[ T_{el} \dot{E}_d = -E_d' - (X_d - X'_d)I_d + E_{dl} \quad (3a) \]

\[ T_{el} \dot{E}_q = -E_q' - (X_q - X'_q)I_q \quad (3b) \]

\[ T_E \dot{E}_\mu = -(K_e + S_e(E_{\mu}))E_{\mu} + V_g \quad (3c) \]

\[ T_r R_i = -R_i + K_e \frac{V}{E_{\mu}} \quad (3d) \]

\[ T_r \dot{V}_h = -V_h + K_e R_i - K_e K_i \frac{V}{E_{\mu}} + K_e (V_{ref} - V) \quad (3e) \]

where the additional states \( E_d' \) and \( E_q' \) are the direct and quadrature axes generator voltages, respectively, \( E_{dl} \) is the equivalent electromotive force (emf) in the direct axis excitation coil, \( V_g \) is the exciter voltage, and \( R_i \) is the rate feedback variable. \( V_{ref} \) saturates when it reaches a predetermined value \( V_{ref, max} \) which can be adjusted to vary the generator’s reactive power capacity. The generator power, \( P_G \), injected into the network is given by:

\[ P_G = V_i \sin(\delta - \theta) \frac{R_e + X_e}{R_e^2 + X_e^2} \left[ (R_e [V_i \cos(\delta - \theta) - E_d'] + X_e [E_q' - V_i \sin(\delta - \theta)]) \right] \]

\[ V_i \cos(\delta - \theta) \frac{[R_e (V_i \sin(\delta - \theta) - E_d') + X_e (E_q' - V_i \sin(\delta - \theta))]}{R_e^2 + X_e^2} \]

Independent inputs within this model (2), (3) are the mechanical power supplied to the generator, \( P_m \), and the generator terminal voltage set point, \( V_{ref} \). Parameter definitions and other variables associated with the flux-decay model can be found in [1, 2].

The proposed reduced model replaces the voltage and excitation dynamics of the flux-decay model (3) with a single state model that captures the basic characteristics of the flux-decay model.

A common approach to generator model simplification is to use a constant emf behind a transient reactance and constant mechanical power during transients [10, 11]. Such an approach neglects the voltage dynamics associated with transient stability and exciter limits [1, 10]. In this paper an intuitive approach is taken to capture voltage dynamics, effects of imposing excitation limits, and steady-state behavior through a simple model.

In the flux-decay model voltage dynamics are created through a transient magnetic flux and voltage excitation. To retain this behavior some representation of an exciter should remain.

Exciter reduction begins with the specification of a dynamic update law for the internal generator voltage \( E \); where \( E \) is analogous to the constant emf in generators lacking voltage excitation. Looking to the flux-decay model to guide the construction of the new state; it is noted that the exciter keeps the generator bus voltage held at a desired value \( V_{ref} \) as long as there is enough reactive power to maintain power balance. When the generator reactive power reaches a maximum, the generator bus voltage \( V \) is allowed to change so that the system remains in power balance \( (V_R \) saturates at this point) and \( V \) will no longer equal \( V_{ref} \).

It is desired that the new state \( E \) perform the same function as the exciter. Therefore \( E \) is chosen such that it maintains a constant bus voltage until its limit is encountered. This is done using the differential equation

\[ \dot{E} = G_e (V_{ref} - V) - T_e E \quad (4a) \]

where \( G_e \) is a gain for the controller allowing the response to be tuned for realistic system responses, and \( T_e \) is a damping coefficient that will keep the controller from having anti-windup problems [12].

Reactive power limits are incorporated by switching the update law for \( E \) in (4a) to the new update law

\[ \dot{E} = G_e (E_{lim} - E) - T_e E \quad (4b) \]

when \( E \) reaches a predetermined maximum value, \( E_{lim} \). \( E_{lim} \) is the maximum allowable internal generator voltage that corresponds to a generator reactive power limit. The generator power, \( P_G \), injected into the network is given by:

\[ P_G = \frac{-V_j R_e}{R_e^2 + X_e^2} + EV \frac{R_e \cos(\delta - \theta) + X_e \sin(\delta - \theta)}{R_e^2 + X_e^2} \]

The constraint on reactive power by limiting \( E \) is similar to the effect of limiting \( V_R \) in the flux-decay generator model. In contrast to similar models presented in [5, 6] the internal voltage \( E \) is not held fixed at the maximum value, but rather is controlled there. Switching between the two differential equations (4a) and (4b) was chosen to make a softer limit on \( E \) in an attempt to avoid chatter in the numerical solution when close to limit.

Combining the exciter (4) with the swing equation (2) yields a new generator model with behaviors chosen to be similar to the flux-decay model. This reduced model will be called the “saturated swing” generator model throughout the rest of the paper.
The test system of figure 1 with power balance equations (1) will now be utilized to compare the behaviors of the two generator models (2), (3) and (2), (4).

3. Steady State Power Transfer Limitations

It has been found that the reactive power limitation of the generator can be a cause of voltage instabilities [5, 6, 9]. Therefore, the bifurcation plots are often drawn based on reactive power loading which will be the approach taken here.

Disregarding any limitations on power input $P_M$, or excitation in the generator; the maximum network capacity can be found. The network itself imposes this limit on the amount of power that can be transferred to the load. This condition occurs at a saddle node bifurcation point which coincides with the maximum transferable power (illustrated in figure 2 for the test system).

![Figure 2. Power transfer limitation with unlimited generation.](image)

An examination of the eigenvalues of the linearization of the system equations (1, 2, and 3) reveals that the solutions on the top half of the curve are stable while the solutions on the bottom half of the curve are unstable [6].

Considering the effects of excitation limits on power system equilibria using the flux-decay generator model it is shown that as the load is increased, the bus voltage decreases (figure 3). For various limiting values of $V_R$ the rate of the voltage decrease and the point at which the bifurcation occurs changes.

![Figure 3. Flux-Decay generator limitations for various $V_{R_{max}}$.](image)

Viewing the stable solutions of the generator limitations and network limitation together (see figure 4) better shows the effect of generator limits.

For the generator limitations shown in figure 4 there are three operating possibilities; unsaturated-stable, saturated-stable, and unstable. Typically a generator will begin operation with an unsaturated-stable solution. As power transfer is increased a limit-induced switching occurs causing the system to become either saturated-stable or unstable (depending on the particular $V_{R_{max}}$). Similarly bifurcation diagrams of the saturated swing model can be created.

![Figure 4. Generator limit shown with network limit](image)

When the saturated swing generator model is considered (with varying limiting values) it is shown that as the load is increased the bus voltage decreases (figure 5). This happens continually until a saddle node bifurcation point occurs and the system loses stability.

![Figure 5. Saturated swing limitations for various $E_{lim}$](image)

Showing only the stable solutions the effect of generator excitation limits can be better viewed (see figure 6).
Just as with the flux-decay model, there are three operating possibilities; unsaturated-stable, saturated-stable, and unstable. As power transfer is increased a limit-induced switching occurs causing the system to become either saturated-stable or unstable (depending on the particular $E_{\text{lim}}$).

For a given state where the stable solution starts on the unlimited system (unsaturated-stable) curve as the load is increased slowly the stable solution will switch to the $E_{\text{lim}}$ (saturated-stable) curve as various limiting values are reached. Just as with the flux-decay model, the point at which these two curves cross is a limit induced switching point and stability will change. As indicated by figures 4 and 6 the two generator models have a similar family of steady-state solutions.

4. Dynamic Power Transfer and Voltage Dynamics

The existence of a stable equilibrium does not guarantee that it will be realized. When dynamics are introduced into the system there is a chance that a disturbance could cause the solution to leave the basin of attraction thereby losing stability. This behavior can be seen in both the flux-decay and saturated swing models.

In the case of a lightly loaded system with $V_{\text{Rmax}}=1.5$ the bus voltages remain stable regardless of the rate of load increase (see sample results in figure 7), up to the saddle-node bifurcation point. The generator model operates in a saturated-stable mode in the figure after the load increase.

However, when the system is heavily loaded with $V_{\text{Rmax}}=3$ it is found that the dynamics introduced through a rapid load change cause the system to become unstable prior to the saddle-node bifurcation as shown in figure 8.

Similar results occur when the saturated swing model is considered for both the lightly and heavily loaded system. First a lightly loaded system ($E_{\text{lim}} = 1$) is considered where the system remains stable regardless of the rate of loading (see sample results in figure 9). This is similar to the case with the flux-decay model results shown in figure 7.

However, for a higher level of loading ($E_{\text{lim}} = 1.6$) if the load is increased quickly, enough energy is introduced into the system to cause the system to leave the basin of attraction as shown in figure 10.
The bifurcation plot shown in figure 6 indicates that with an $E_{lim}$ of 1.6 the maximum achievable steady state reactive load should be approximately 10pu. However, the addition of dynamics causes a loss of stability prematurely; just as with the flux-decay model response shown in figure 8.

To further compare the voltage dynamics of the generator models a step response is examined. For this study $V_{ref}$ is increased one percent and the dynamic response of the voltage deviation from its equilibrium is recorded. The step response of the flux-decay model is shown in figure 11 where it can be seen that the time to the peak voltage occurs about 7 seconds after the input change. The oscillations caused by the input change subside over about 40 seconds and have a settling time to within 2% of the final value of 15 seconds, with a steady state error better than 0.0005 per-units.

Another important benchmark is to see how the saturated swing generator responds to a step increase in a reference voltage. Again, the generator parameter $V_{ref}$ is increased one percent and the resulting generator response is shown in figure 12. For this model the time to the peak voltage is about 7 seconds after the input change. The oscillations caused by the input change subside over about 30 seconds and have a settling time to within 2% of the final value of 18 seconds, with a steady state error better than 0.001 per-units.

To further compare the generator models, a larger system is considered. The system chosen is the WSCC 9-bus test case (figure 14). System parameters for this test case can be found in [2].
For the comparison of the two models an extra line was added from bus 4 to 6. This line was tripped after 1 second of simulation so that dynamics could be introduced into the system without changing the structure of the network. The resulting network voltage dynamics are shown for both saturated swing and flux decay generator models. For clarity only two bus voltages (busses 6 and 8) are shown in figure 15.

From the figure it is apparent that the flux decay and saturated swing models both yield the same steady-state voltages prior to the disturbance. After the disturbance the saturated swing model has a steady-state value of approximately 0.0004 per-units less than the flux-decay model. Rise time and peak voltage for both models appear very similar and the settling time for the saturated swing model is only slightly quicker than that of the flux-decay model.

5. Conclusion

A simple generator model was presented for power system studies, and in particular, voltage stability studies in the presence of excitation limits. Comparison with a well-known flux-decay model shows similar evolution in equilibria, response to load variations, and transient behavior. These results indicate that the reduced model could replace more complicated models when simplicity is required without the loss of exciter dynamics and limits.

References: