Parameter Estimation of Hybrid Dynamical Systems With Delayed Switching

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Abstract

In this paper, we address parameter estimation for hybrid dynamical systems with state-dependent events and time-delayed parametric switching. Parameter estimation of hybrid systems can aid in system identification, model refinement, and contingency and stability analysis in various fields, including chemical, electrical, and power systems engineering. We discuss parameter estimation for hybrid systems and introduce the concept of time-delay in switching events. Hybrid systems are difficult to analyze, and complicate the calculation of trajectory sensitivities used in gradient estimation methods. We compute the proper jump conditions for these trajectory sensitivities when state-dependent, time-delayed switching occurs. While this study is fairly general to hybrid systems, it is presented with an emphasis on electric power systems, whose continuous dynamics consist of power generation, transmission, and consumption, and whose discrete dynamics are governed by protection logic devices.

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1 Introduction

It is typical that many details of a modelled system will not be known with sufficient accuracy for a particular application. Parameter estimation techniques use system measurements to reduce model uncertainty, and are used in such areas as system identification and model refinement. Such refinement can be important in many applications, especially for dynamic systems. Accurate parameters for dynamic models help to ensure reliable simulations, which can provide valuable insight regarding transient phenomena ([10]). Parameter accuracy can also be critically important in event reconstruction and system diagnostics ([4]), as well as system stability studies ([8],[9],[15]). Parameter estimation can also be used for model recalibration in systems which exhibit parametric trends over time ([11]).

In this paper, we address parameter estimation for hybrid dynamical systems with time-delayed discrete events. A hybrid system is composed of interconnected continuous and discrete systems, whose states evolve through continuous dynamics and discrete switching behavior ([1]). Discrete events are generally state-dependent, i.e., they are triggered based on the current behavior of the system. We will consider problems where the continuous dynamics are governed by differential algebraic equations (DAEs). For the electric power system models of interest to the authors, algebraic equations govern the fast power-balance dynamics on the grid, and the slow internal dynamics of the loads and generators are governed by differential equations.

A discrete event is triggered when certain criteria are met. In a time-delayed event, this triggering sets a future switching time. If certain cancellation criteria are met before this time, no switching occurs; otherwise, switching occurs as scheduled. This type of discrete system dynamics is useful in describing the behavior of protective switching devices, which may delay activation in order to give the system time to return to an acceptable state. This can help to prevent "chattering" and overreactions to small disturbances, which can negatively affect system stability ([13]).

We consider a gradient-based optimization method for our parameter estimation problem. Such methods requires the computation of state sensitivities with respect to the unknown parameters; in a dynamic setting these must be computed along the system trajectory ([5]). When switching occurs in hybrid systems, proper jump conditions must be computed for the trajectory sensitivities ([6]). The jump conditions depend upon the switching mechanism as well as both the pre- and post-switch systems. To simplify the analysis, we only consider parametric switching. While this is not completely general, it sufficiently describes the systems of interest.

In section 2 we describe a hybrid DAE system with delayed switching. In section 3 we present parameter estimation and trajectory sensitivities, and address the computation of sensitivity jump conditions when switching occurs. In section 4 we consider a basic transmission-level DAE model for electric power systems, and present analysis results for the WSCC 9-bus test grid.

2 Delayed-Switching Hybrid Dynamical Systems

A hybrid dynamical system with time-delayed switching is a set of coupled continuous- and discrete-time systems, each with a set of continuous states, discrete states (parameters), and switching procedures. A switching condition has an associated triggering criterion, time delay, cancellation criterion, and state-dependent parametric switching function. Algebraic equations and states may be included in the continuous system. Consider the differential-algebraic system given by

$$\dot{x} = f(x, y, u; \lambda) \tag{1}$$

$$0 = g(x, y, u; \lambda) \tag{2}$$

where $f : \mathbb{R}^{n+m+p+v} \to \mathbb{R}^n$, $g : \mathbb{R}^{n+m+p+v} \to \mathbb{R}^m$, $x(t) \in \mathbb{R}^n$ is the continuous states at time t, $y(t) \in \mathbb{R}^m$ is the algebraic states, $u(t) \in \mathbb{R}^v$ is the inputs (which are assumed to be smooth), and the set of system parameters is $\lambda \in \mathbb{R}^p$. We assume that a set of outputs

$$z = m(x, y, u; \lambda) \tag{3}$$

is given by the system, where $z \in \mathbb{R}^o$ and $m : \mathbb{R}^{n+m+p+v} \to \mathbb{R}^o$. It is assumed that for all $u(t) \in C \subset \mathbb{R}^v$ and $(x_0, y_0, \lambda_0) \in D \subset \mathbb{R}^{n+m+p}$, functions f and g are sufficiently smooth for the system to be well posed. As $u(t)$ is smooth and parameter-independent, it does not affect the following analysis and is suppressed from here on. It is further assumed that the partial of g with respect to y, g_y, is nonsingular, making the system an index 1 DAE. Initial conditions are given by

$$\begin{aligned} x(0) &= x_0 \\ y(0) &= y_0 \\ \lambda &= \lambda_0 \end{aligned} \tag{4}$$

The discrete system is described through a finite set of possible switching procedures. The jth switching procedure is characterized by a set of functions $\{s_j(x, y; \lambda), c_j(x, y; \lambda), h_j(\lambda)\}$ (triggering, cancellation, and switching functions, respectively) and a time delay $\Delta_j \geq 0$. A zero-crossing of $s_j(x, y; \lambda)$ along the trajectory $\{x(t), y(t)\}$ triggers the event. When an event is triggered at time τ, the switching time $\tau + \Delta_j$ is set. If $c_j(x, y; \lambda)$ does not cross zero before this time, one or more parameters are switched through $\lambda \leftarrow h_j(\lambda)$. Suppressing the index j, assume that the event is not cancelled by a zero-crossing of c; then the parametric switching occurs at time $\tau + \Delta$. Let the switched system be given by

$$\dot{x}^{+} = f\left(x^{+}, y^{+}; \lambda^{+}\right) \tag{5}$$

$$0 = g\left(x^{+}, y^{+}; \lambda^{+}\right) \tag{6}$$

with the initial conditions satisfying

$$x^{+}(\tau + \Delta) = x(\tau + \Delta) \tag{7}$$

$$\lambda^{+} = h(\lambda) \tag{8}$$

$$g(x^{+}(\tau + \Delta), y^{+}(\tau + \Delta); \lambda^{+}) = 0 \tag{9}$$

where (9) defines $y^{+}(\tau + \Delta)$.

The system can experience more than one switching event over time. The system trajectory is piecewise smooth, with possible discontinuities at switching times. Across a switching time, x is assumed to be continuous, (7), the parameters switch, (8), and the algebraic states may undergo a discontinuous jump in order to provide consistent initial conditions (9) for the post-switch DAE (5-6).

3 The Parameter Estimation Problem

Given measurements of the system output $z = m(x, y; \lambda)$, we wish to estimate some subset, $\lambda_e \in \mathbb{R}^w$, of the initial parameter values λ_0. Assume the system output is measured at times $\{t_1,\ t_2,\ \ldots\ t_q\}$, and let these measurements be given by $\{\hat{z}(t_1),\ \hat{z}(t_2),\ \ldots\ \hat{z}(t_q)\}$. We define the best estimate to be the value of λ_e which minimizes a given objective function

$$W(\lambda_e) : \{r_1(\lambda_0),\ r_2(\lambda_0),\ \ldots\ r_q(\lambda_0)\} \rightarrow \mathbb{R} \tag{10}$$

where $r_i(\lambda_0) = z(t_i, \lambda_0) - \hat{z}(t_i)$, and $z(t_i, \lambda_0)$ is obtained through simulation of the system with the current estimate λ_e. We assume that $W(\lambda_e)$ has a local minimum λ_e^*, at which the gradient $W_{\lambda_e}(\lambda_e^*)$ vanishes and the Hessian $W_{\lambda_e \lambda_e}(\lambda_e^*)$ is positive semidefinite. We consider iterative gradient methods to minimize the quadratic objective function $W(\lambda_e) = \frac{1}{2} \sum_{i=1}^{q} r_i(\lambda_0)^2$. Such methods iteratively compute a parameter update $\Delta\lambda_e$ using the gradient W_{λ_e}, and apply the update through $\lambda_e \leftarrow \lambda_e + \Delta\lambda_e$. We choose the Gauss-Newton method, where $\Delta\lambda_e$ is computed by solving

$$\left(e_{\lambda_e}^T e_{\lambda_e}\right) \Delta\lambda_e = -e_{\lambda_e}^T e \tag{11}$$

where e is the column vector (r_i), $\quad i = 1 \ldots q$. More sophisticated optimization schemes can be used; however, all gradient schemes call for the computation of the sensitivities $\{z_{\lambda_e}(t_i, \lambda_0)\}$. Without loss of generality, we now discuss the computation of sensitivities with respect to the full set of initial parameters λ_0, with the understanding that only the sensitivities pertaining to λ_e are actually used for parameter estimation.

The sensitivities can be obtained by solving the sensitivity equations

$$\begin{aligned} \dot{x_{\lambda_0}} &= f_x x_{\lambda_0} + f_y y_{\lambda_0} + f_\lambda \lambda_{\lambda_0} && (12) \\ 0 &= g_x x_{\lambda_0} + g_y y_{\lambda_0} + g_\lambda \lambda_{\lambda_0} && (13) \end{aligned}$$

obtained by taking the gradient of (1-2) with respect to λ_0 and reversing the order of differentiation in (12). Subscripts denote partial differentiation. The sensitivity equations form a linear time-varying DAE. Differentiating (4) with respect to λ_0 gives initial conditions

$$\begin{aligned} x_{\lambda_0}(0) &= 0 \\ y_{\lambda_0}(0) &= 0 \\ \lambda_{\lambda_0} &= \mathbf{I}_{p \times p} \end{aligned} \tag{14}$$

We assume conditions on f and g to guarantee that the problem (12-14) is well posed. The output sensitivities satisfy

$$z_{\lambda_0} = m_x x_{\lambda_0} + m_y y_{\lambda_0} + m_\lambda \lambda_{\lambda_0} \tag{15}$$

After an event, the sensitivity equations corresponding to (5-6) are given by

$$\dot{x}^+_{\lambda_0} = f_{x^+} x^+_{\lambda_0} + f_{y^+} y^+_{\lambda_0} + f_{\lambda^+} \lambda^+_{\lambda_0} \tag{16}$$

$$0 = g_{x^+} x^+_{\lambda_0} + g_{y^+} y^+_{\lambda_0} + g_{\lambda^+} \lambda^+_{\lambda_0} \tag{17}$$

The following sensitivity jump conditions provide the correct initial conditions for (16-17).

3.1 Sensitivity Jump Conditions

Consider a switching event of the type described in Section 2. We assume that our triggering function s is smooth, and that $\frac{ds}{dt}|_\tau$ is nonzero, so that $s = 0$ has a locally unique solution $t = \tau(\lambda_0)$ along the system trajectory. When $c \neq 0$ in the time interval $(\tau, \tau + \Delta]$, the switching occurs at time $\tau(\lambda_0) + \Delta$ through $\lambda^+ = h(\lambda)$, giving us a new system (5-9) with new sensitivity equations (16-17). We must compute consistent initial conditions $x^+_{\lambda_0}(\tau + \Delta)$, $\lambda^+_{\lambda_0}(\tau + \Delta)$, and $y^+_{\lambda_0}(\tau + \Delta)$ for these equations. In order to do so, we assume that the switching function $h(\lambda)$ is smooth so that $\lambda^+_{\lambda_0}$ is well defined. Also note that $c|_{\tau+\Delta} \neq 0$, so that $x^+_{\lambda_0}(\tau + \Delta)$ is well defined. We use the notation $f^+(t)$ to denote $f(x^+(t), y^+(t), \lambda^+)$ and $f(t)$ to denote $f(x(t), y(t), \lambda)$, and similarly for g.

Theorem 3.1. *At a switching event enacted through (7 - 9), the following jump conditions hold*

$$x^+_{\lambda_0} = x_{\lambda_0} + \left(f - f^+\right) \tau_{\lambda_0} \tag{18}$$

$$\lambda^+_{\lambda_0} = h_\lambda \lambda_{\lambda_0} \tag{19}$$

$$y^+_{\lambda_0} = -\left(g^+_y\right)^{-1} \left(g^+_x x^+_{\lambda_0} + g^+_\lambda \lambda^+_{\lambda_0}\right) \tag{20}$$

where all quantities are evaluated at time $\tau + \Delta$, except τ_{λ_0} which is given at time τ by

$$\tau_{\lambda_0} = -\frac{\left(s_x - s_y \left(g_y\right)^{-1} g_x\right) x_{\lambda_0} + \left(s_\lambda - s_y \left(g_y\right)^{-1} g_\lambda\right) \lambda_{\lambda_0}}{\left(s_x - s_y \left(g_y\right)^{-1} g_x\right) f} \tag{21}$$

Proof. Since s is smooth and $\frac{ds}{dt}|_\tau \neq 0$, $s = 0$ has a unique solution $t = \tau(\lambda_0)$ which is smooth in λ_0, so that from (7) we have

$$\frac{d}{d\lambda_0} \left(x^+(t, \lambda_0)\Big|_{\tau(\lambda_0)+\Delta} \right) = \frac{d}{d\lambda_0} \left(x(t, \lambda_0)\Big|_{\tau(\lambda_0)+\Delta} \right)$$

Since $c|_{\tau+\Delta} \neq 0$, the jump condition $x^+_{\lambda_0}(\tau+\Delta)$ is well defined. Applying the chain rule, at time $\tau+\Delta$ we find

$$\begin{aligned} x^+_{\lambda_0} + (x^+_t)(\tau+\Delta)_{\lambda_0} &= x_{\lambda_0} + (x_t)(\tau+\Delta)_{\lambda_0} \\ x^+_{\lambda_0} &= x_{\lambda_0} + \left(x_t - x^+_t\right)(\tau+\Delta)_{\lambda_0} \\ &= x_{\lambda_0} + \left(f - f^+\right)\tau_{\lambda_0} \end{aligned}$$

To compute τ_{λ_0}, we consider the equation $s = 0$ in the pre-switch system at time τ. Taking its λ_0 gradient and applying the chain rule, we get

$$\begin{aligned} 0 &= \frac{d}{d\lambda_0}\left(s(x(t,\lambda_0), y(t,\lambda_0), \lambda(\lambda_0))\big|_{\tau(\lambda_0)} \right) \\ 0 &= s_x\left(x_{\lambda_0} + x_t\tau_{\lambda_0}\right) + s_y\left(y_{\lambda_0} + y_t\tau_{\lambda_0}\right) + s_\lambda\left(\lambda_{\lambda_0}\right) \\ &= \left(s_x - s_y\left(g_y\right)^{-1} g_x\right)\left(x_{\lambda_0} + f\tau_{\lambda_0}\right) + \left(s_\lambda - s_y\left(g_y\right)^{-1} g_\lambda\right)\lambda_{\lambda_0} \end{aligned}$$

Solving for τ_{λ_0}, we get (21) which completes the jump condition for $x^+_{\lambda_0}$. Differentiating (8) with respect to λ_0 gives the jump condition for $\lambda^+_{\lambda_0}$ (19). Finally, having calculated $x^+_{\lambda_0}$ and $\lambda^+_{\lambda_0}$, we differentiate (9) with respect to λ_0 and solve for $y^+_{\lambda_0}$, giving (20), which completes the derivation of (18-21). □

These jump conditions can be modified to apply to other situations. For instance, consider the problem of estimating initial parameter values during a switching event that occurs due to an exogenous input at a given time–for instance, a power transmission line tripping due to a lightning strike. In this case we can redefine τ to be the given switching time and set $\Delta = 0$. Since τ is system-independent, $\tau_{\lambda_0} = 0$. Our sensitivity jump conditions (18 - 20) become

$$x^+_{\lambda_0} = x_{\lambda_0} \tag{22}$$

$$\lambda^+_{\lambda_0} = h_\lambda \lambda_{\lambda_0} \tag{23}$$

$$y^+_{\lambda_0} = -\left(g^+_y\right)^{-1}\left(g^+_x x^+_{\lambda_0} + g^+_\lambda \lambda^+_{\lambda_0}\right) \tag{24}$$

In other circumstances, the switching mechanism $h(\lambda)$ may not be known exactly, and it may be desireable to estimate some subset of the post-switch parameter values λ^+. For example, the power signature of a load could suddenly change, signifying an event, but it could be unknown how the internal load characteristics changed to cause the event. Whether the switching condition is state-dependent, or occurs at a fixed time (as above), all sensitivities with respect to λ^+ are zero previous to the switching time, and clearly $\tau_{\lambda^+} = 0$. The sensitivity equations are given by

$$\dot{x}^+_{\lambda^+} = f_{x^+} x^+_{\lambda^+} + f_{y^+} y^+_{\lambda^+} + f_{\lambda^+} \lambda^+_{\lambda^+} \quad (25)$$

$$0 = g_{x^+} x^+_{\lambda^+} + g_{y^+} y^+_{\lambda^+} + f_{\lambda^+} \lambda^+_{\lambda^+} \quad (26)$$

and start from the switching time with initial conditions

$$x^+_{\lambda^+} = 0 \quad (27)$$

$$\lambda^+_{\lambda^+} = \mathbf{I}_{p \times p} \quad (28)$$

$$y^+_{\lambda^+} = -\left(g_{y^+}\right)^{-1} \left(g^+_{\lambda}\right) \quad (29)$$

It is also possible to calculate the sensitivity of the post-switch system with respect to the switching time itself. Similarly, we can compute sensitivities to parameters that are unique to the triggering function s, such as event threshholds. Thus, trajectory sensitivities can be used to estimate not only parameters of the continuous system, but also event attributes and switching device characteristics. Applications could include predictive simulation, supplementing and error-checking sensor measurements, and post-event diagnostics.

4 Power System Examples

We use a hybrid DAE system to model an electric power grid at the transmission level. The grid is modelled as a set of busses (nodes), which are interconnected by transmission lines and can each have any number of attached generators and loads. The power generation/consumption dynamics of these generators and loads are modelled through differential equations, while the much faster transmission dynamics are assumed to be instantaneous and are modelled through algebraic equations. We also consider the behavior of protection logic devices, which monitor certain aspects of the grid's behavior and, if certain criteria are met, cause switching events in an attempt to stabilize the system or protect equipment. Such a model provides insight into the major dynamics of the grid at a sub-second time resolution, and can be used in studies of system stability, contingency behavior, and other phenomena where dynamic behavior is of importance.

The WSCC 9-bus test system (Figure 1) was used for these examples. In absence of real data, data for parameter estimation is fabricated through prior simulation. A Matlab research code

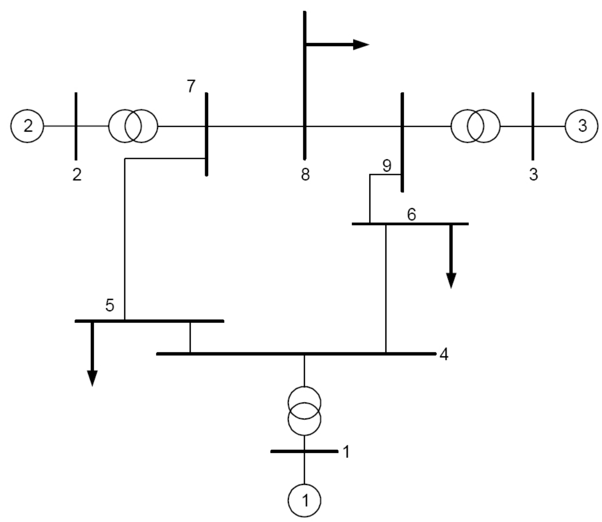

Figure 1: The WSCC 9-bus test system

is used to generate all trajectories and sensitivities for estimation. The equations are solved with the Matlab function Ode15s, a multi-order NDF method with variable timestepping, due to its event-handling capability and efficient, accurate solution of stiff DAEs ([14]). In other programming languages, any efficient method with stiff decay, such as those based on the backward difference formulas, would likely suffice (a discussion of such methods is given in [2]).

Consistent initial conditions for the system are computed by loadflow solution and subsequent calculation of consistent initial values for the differential states. As this process is parameter-dependent, equations (4) and (14) do not apply, and consistent initial conditions for the sensitivity equations (12-13) must be calculated in a different manner. We therefore employ numerical differentiation techniques, as the computation of the system initial conditions for multiple parameter values is sufficiently inexpensive.

For this study, we use fourth-order flux-decay generator models, as described in ([12]). The load dynamics are governed by the exponential recovery model as described in ([3],[7]), which is an aggregate load model designed to fit empirical measurements of load powers during step/ramp disturbances in bus voltage. The transmission dynamics are given by algebraic power balance equations which are a direct result of Kirchoff's laws ([12]). Studies are performed in a timescale of several seconds, consistent with the subsecond-to-minutes timescale of the load and generator models. As both the load model and power balance model are directly relevant to the examples, they are presented below.

Power Balance Equations

In the following description of power balance at bus j, P_{Gj} and Q_{Gj} are the sum of real and reactive generator powers, respectively. P_{Lj} and Q_{Lj} are defined similarly for loads, and are assumed to have the opposite sign convention of the generator powers. States V_j and θ_j are the amplitude and phase of the sinusoidal bus voltage. G_{jk} and B_{jk} are the conductance and susceptance of the transmission line between busses j and k, and are zero if no line exists. The tilde ($\tilde{}$) denotes shunt values, and N is the set of all bus indices.

$$
\begin{aligned}
0 &= P_{Gj} + P_{Lj} - \sum_{k \in N \setminus \{j\}} \left((G_{jk} + \frac{1}{2}\tilde{G}_{jk})V_j^2 - G_{jk}V_jV_k \cos(\theta_j - \theta_k) - B_{jk}V_jV_k \sin(\theta_j - \theta_k) \right) \\
0 &= Q_{Gj} + Q_{Lj} - \sum_{k \in N \setminus \{j\}} \left(-(B_{jk} + \frac{1}{2}\tilde{B}_{jk})V_j^2 + B_{jk}V_jV_k \cos(\theta_j - \theta_k) - G_{jk}V_jV_k \sin(\theta_j - \theta_k) \right)
\end{aligned}
$$

Exponential Recovery Load Equations

At a given load, the power profile is defined by the following

$$
\begin{aligned}
P_L &= \frac{x_p}{T_p} + P_0 \left(\frac{V}{V_0}\right)^{\alpha_t} \\
Q_L &= \frac{x_q}{T_q} + Q_0 \left(\frac{V}{V_0}\right)^{\beta_t}
\end{aligned}
$$

where P_0 and Q_0 are the nominal real and reactive power and V_0 is the nominal bus voltage. The parameters T_p, T_q, α_s, β_s, α_t, and β_t describe the time-dynamics of the load, and are generally fit to data. States x_p and x_q are defined by the following:

$$
\begin{aligned}
\dot{x}_p &= \frac{-x_p}{T_p} + P_0 \left(\left(\frac{V}{V_0}\right)^{\alpha_s} - \left(\frac{V}{V_0}\right)^{\alpha_t} \right) \\
\dot{x}_q &= \frac{-x_q}{T_q} + Q_0 \left(\left(\frac{V}{V_0}\right)^{\beta_s} - \left(\frac{V}{V_0}\right)^{\beta_t} \right)
\end{aligned}
$$

4.1 Line estimation after thermal line trip

In this model, we consider a threshhold P_{max} for injected power into a line between busses j and k. If the injected power on either end of the line exceeds this threshhold for more than a given time delay Δ, the line is removed from service. Put into the framework of section 2, this time-delayed hybrid switching event can be described as follows. The triggering function s is given by

$$s = max(\left|S_{jk}\right|, \left|S_{kj}\right|) - P_{max}$$

where the complex power injected into line j-k is given by

$$S_{jk} = V_j e^{i\theta_j} \left(\left(V_j e^{i\theta_j} - V_k e^{i\theta_k} \right) (G_{jk} + iB_{jk}) + \frac{1}{2} V_j e^{i\theta_j} \left(\tilde{G}_{jk} + i\tilde{B}_{jk} \right) \right)^*$$

and where $(\ )^*$ denotes complex conjugation and $i = \sqrt{-1}$. If the power dips below the triggering threshhold between time τ and $\tau + \Delta$, the switch is cancelled; thus $c = \pm s$ describes the cancellation criterion. The switching function is given by $h(\lambda_l) = 0$ if l is the index of G_{jk}, B_{jk}, $\tilde{G}_{jk}$, or $\tilde{B}_{jk}$, and $h(\lambda_l) = \lambda_l$ otherwise. The derivatives of s with respect to the states and parameters depend on whether $|S_{jk}|$ or $|S_{kj}|$ exceeded the threshhold. Assuming $|S_{jk}|$ exceeded the threshhold:

$$s_\lambda = \left|S_{jk}\right|_\lambda = \frac{Re(S_{jk})Re(S_{jk_\lambda}) + Im(S_{jk})Im(S_{jk_\lambda})}{\left|S_{jk}\right|}$$

The derivatives of S_{jk} with respect to the relevant states and parameters are given as follows:

$$\begin{aligned}
S_{jk_{V_j}} &= V_j \left(2(G - iB) + \tilde{G} - i\tilde{B} \right) - V_k e^{i(\theta_j - \theta_k)} (G - iB) \\
S_{jk_{V_k}} &= -V_j (G - iB) e^{i(\theta_j - \theta_k)} \\
S_{jk_{\theta_j}} &= -iV_j V_k (G - iB) e^{i(\theta_j - \theta_k)} \\
S_{jk_{\theta_k}} &= iV_j V_k (G - iB) e^{i(\theta_j - \theta_k)} \\
S_{jk_B} &= -iV_j \left(V_j - V_k e^{i(\theta_j - \theta_k)} \right)
\end{aligned}$$

For this example (results in Figure 2), the mechanical power at generator 1 was smoothly increased to 125% of nominal between $t = 28\ s$ and $t = 30$ s, and then decreased to nominal by $t = 32\ s$. The transformer between busses 1 and 4 is modelled as a line, denoted line 1-4 from here on. P_{max} at line 1-4 was lowered to a value of .85 (typical values around 3.0) to make it susceptible to thermal line tripping. A thermal line trip at line 1-4 is triggered at $t \approx 30.14\ s$, and enacted at $t \approx 30.15\ s$ ($\Delta = 0.01$). The voltage at bus 6 was sampled at a rate of 10 Hz starting at $t = 30.5\ s$. From this data we are able to estimate the pre-switch value of the susceptance at line 1-4, converging in 5 iterations to a value of -17.3604 (from an initial guess of -12).

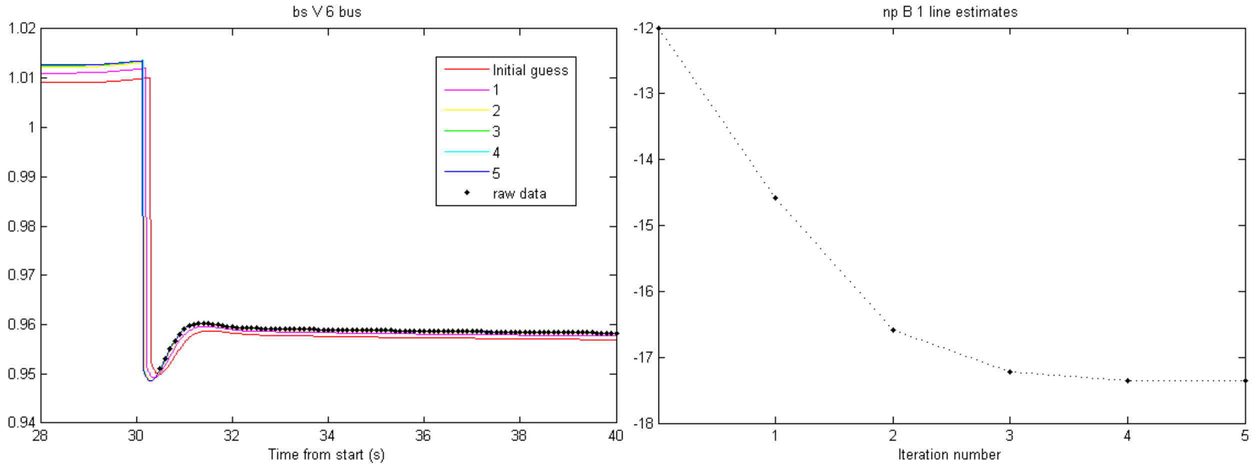

Figure 2: Convergence of trajectories and parameter estimates

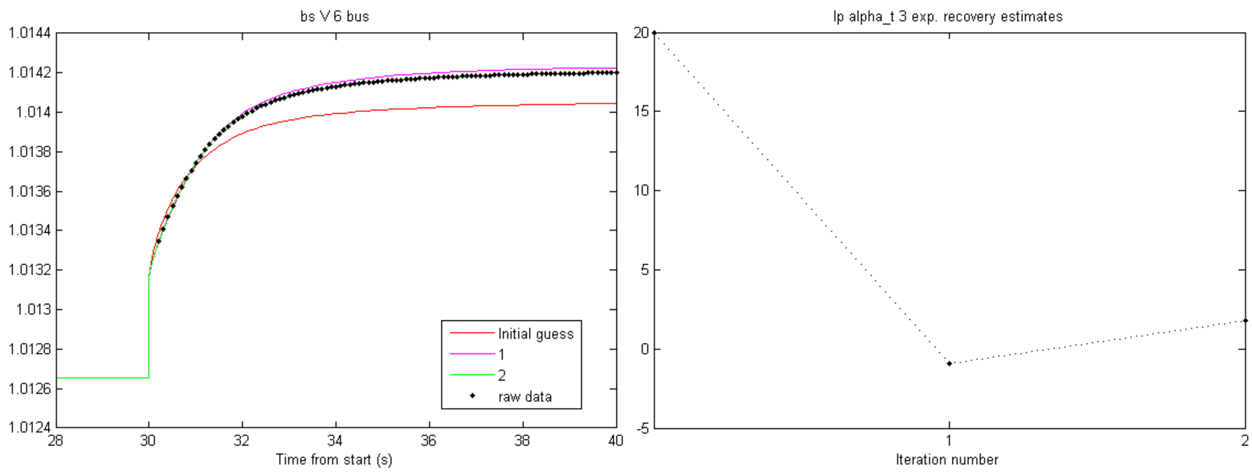

Figure 3: Convergence of trajectories and parameter estimates

4.2 Estimation of α_t after a step change at a different load

For this example, which illustrates estimation during an event at a fixed time, we estimate the parameter α_t at load 3 after a step change at load 1. This step change was an instantaneous reduction of P_0 to 90% of its original value at time $t = 30\ s$. The voltage at bus 6 was again measured at a rate of 10 Hz, starting at $t = 30.2\ s$. Starting with an initial guess of 20, the method converged to a value of 1.796 in 2 iterations. Results are presented in Figure 3.

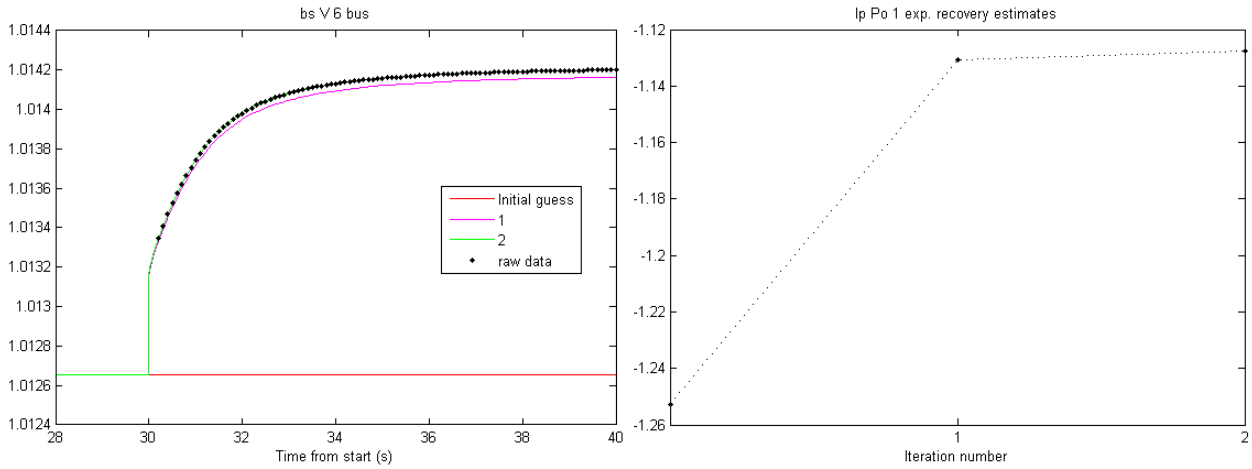

Figure 4: Convergence of trajectories and parameter estimates

4.3 Estimation of post-switch P_0 after load step change

This example illustrates estimation of post-event parameter values during an event at a fixed time. For this example, we used the same event as above to create the data. However, we wished to estimate the post-switch value of P_0 at load 1 itself, assuming the event time was known. Using the pre-switch value (-1.2527) as an initial guess, the method converged to a value of -1.1275 in two iterations. Note that this is indeed very close to 90% of the pre-switch value. Results are shown in Figure 4.

5 Conclusion

Parameter estimation of hybrid systems aids in system identification and model refinement, and improves accuracy in simulation-based system studies. We addressed parameter estimation for hybrid dynamical systems with state-dependent events and time-delayed parametric switching. The calculation of trajectory sensitivities (used in gradient methods for parameter estimation) is complicated by such switching. We computed the proper jump conditions for the trajectory sensitivities when time-delayed, state-dependent switching occurs; we also showed the derivation of jump conditions in the case of fixed-time events or for sensitivities to post-switch parameter values. In the examples, it can be seen how these jump conditions allow estimation of electric power system parameters after such switching events. Various other event types for power systems can be modelled, including load sheds and state saturation.

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